Phys 410
Spring 2013
Lecture #7 Summary
6 February, 2013

Angular momentum is a measure of the difficulty of bringing a rotating object to rest. One can define the angular momentum of a single particle, relative to an arbitrarily chosen origin as $\vec{\ell} = \vec{r} \times \vec{p}$, where $\vec{r}$ is the coordinate of the particle and $\vec{p}$ is its linear momentum. We showed that the time-derivative of the angular momentum is $\dot{\vec{\ell}} = \vec{r} \times \vec{F} = \vec{\Gamma}$, where we have defined the torque $\vec{\Gamma}$. Torque is an influence that causes angular acceleration, just as force is an influence that causes linear acceleration. Note that the angular momentum and torque must be calculated using the same origin.

When a planet orbits a star, it does so under the influence of gravity. Gravity exerts no torque on the planet, hence its angular momentum is conserved. We showed that the angular momentum is $\vec{\ell} = mr^2\dot{\phi}\,\hat{z}$, where $\hat{z}$ is the direction perpendicular to the plane formed by the position and momentum vectors of the planet, with the origin in the center of the star. From this result one can show that the position vector of the planet sweeps out equal areas in equal times, $\dot{A} = \frac{1}{2}\frac{|\vec{\ell}|}{m}$, known as Kepler's second law of motion.

One can write the total angular momentum of a system of particles as $\vec{L} = \sum_{\alpha=1}^{N} \overrightarrow{r_\alpha} \times \overrightarrow{p_\alpha}$. The time rate of change of the total angular momentum vector is equal to the net external torque acting on the system: $\dot{\vec{L}} = \sum_{\alpha=1}^{N} \overrightarrow{r_\alpha} \times \overrightarrow{F_{ext}} = \Gamma^{ext}$. This is Newton's second law of rotational motion for extended multi-particle systems. Its derivation assumes 1) all the internal forces are central in nature – they act along the line between the particles, and 2) the internal forces obey Newton's third law.

It is often convenient to write the angular momentum of a rigid body in terms of the moment of inertia as: $L_z = I_z\omega$, where the axis of rotation coincides with the z-axis and the object has angular velocity ω. You will show for homework that $I_z = \sum_{\alpha=1}^{N} m_\alpha(x_\alpha^2 + y_\alpha^2)$.

Finally, we considered the problem (Taylor 3.35) of a solid disk rolling down an inclined plane, and solved for the acceleration of the center of mass.